# A New Microwave Phase Equalizer Network

**Abstract—This correspondence describes a new microwave allpass phase equalizer network using directly connected TEM mode transmission lines. Networks of this type have found recent application in wideband radar systems employing microwave matched filters.**

**In the network described, transmission lines are interconnected to closely approximate a periodic repetition of the lowpass transfer function $H(p)=(p-a)/p+a$, where $p$ is the complex frequency variable $p=\sigma+j\omega$. A synthesis technique is presented which permits the designer to arbitrarily locate the pole-zero pair of the transfer function in the left and right half-plane, respectively.**

**The correspondence concludes with an illustrative filter design. A network meeting the design specifications is fabricated in the shop and its response is evaluated in the laboratory using time-domain reflectometer techniques. The experiment results are shown to be in close agreement with the theory.**

## Introduction

The purpose of this correspondence is to describe a new microwave allpass phase equalizer network using directly connected (i.e., as opposed to coupled) TEM mode transmission lines.[1] The system function of this network is given by

$$H(p) = A \prod_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \frac{p - j(2k-1)\omega_0 - a}{p - j(2k-1)\omega_0 + a} \qquad (1)$$

where $A$ is a constant, and $p$ is the complex frequency variable $p=\sigma+j\omega$, $H(p)$ has a pole-zero pair located at $p=j(2k-1)\omega_0 \mp a$ in the left and right half $p$ plane, respectively. The device is "resonant" at odd multiples of the resonant frequency $\omega_0$. Networks of this type can be synthesized by adjusting their transient behavior, that is, by selecting the proper system function ($p$ domain) or impulse response ($t$ domain).

Recent interest in microwave matched filter design[2,3] has stimulated the development of a "restricted" class of allpass phase equalizer networks employing, for example, a four-port ring hybrid or "rat race."[4] The ring is connected so that two of its ports are terminated in dual impedances $Z$ and $1/Z$ ohms; the input and output ports are matched to the source and load terminations, respectively. This network is restricted in the sense that its "allpass" properties are valid only within the frequency range that the rat race performs as a hybrid junction. The time delay versus frequency properties of this device are a sensitive function of the impedance $Z$.

Another approach to the problem and the one adopted here is to synthesize the desired response directly in the complex $p$ plane using only TEM mode connected lines. It has been shown that an appropriate connection of TEM mode transmission line results in a system function which is meromorphic with a periodic pole-zero pattern.[5] If the region of interest is restricted to the behavior around one such pole-cluster, a satisfactory wideband approximation results. (*Note:* Here a "cluster" could span an octave or more.) The technique presented permits the designer to choose the location of poles or zeros independently. A simple network whose system function consists of a periodic repetition of poles in the LHP is described. This is followed by a description of a network exhibiting only transmission zeros in the right half-plane (RHP). The networks are placed in tandem separated first by ideal isolators and later by attenuators. The system parameters are chosen so that the pole-zero pairs exhibit mirror symmetry about the $j\omega$ axis. A theoretical analysis of the network is presented, concluding with the illustrative design of a filter which is experimentally evaluated in the laboratory.

## Theoretical Details

We first investigate the condition required to achieve a periodic repetition of poles in the left half-plane (LHP). Consider the network shown in Fig. 1(a) consisting of a TEM mode transmission line having a characteristic impedance of $R_0$ ohms and terminated in one ohm. The network is fed by a 2 volt-second impulse source having an internal impedance of one ohm. It can be shown that the system function $H_1(z)$ for this two-port network is given by

$$H_1(z) = \frac{E_{\text{out}}}{E_{\text{in}}} = \frac{(1-\Gamma^2)z}{1-\Gamma^2 z^2} \qquad (2)$$

where

$$\Gamma \equiv \frac{R_0 - 1}{R_0 + 1},$$

$$z = e^{-p\tau},$$

$$\tau = \frac{L(\text{length of the line})}{c(\text{the speed of light in the medium})}.$$

The function described by (2) has a pole at the points

$$p = \frac{1}{2\tau}\ln \Gamma^2 \pm j\frac{\pi n}{\tau} \qquad n = 0, 1, 2, \cdots \qquad (3)$$

as shown in Fig. 1(b).

Next we investigate a network for achieving a periodic repetition of zeros in the left and right half-planes. Consider the network shown in Fig. 2(a) consisting of a TEM mode line having a characteristic impedance of one ohm and feeding a $\frac{1}{2}$ ohm stub of length $L$ terminated in a resistance $r$ ohms. It is not difficult to show that the impulse response of this network is given by

$$h_2(t) = \frac{1}{2}\delta\left(t - \frac{L}{c}\right) + \frac{\gamma\delta}{2}\left(t - \frac{3L}{c}\right) = \frac{1}{2}\{\delta(t-\tau) + \gamma\delta(t-3\tau)\} \qquad (4)$$

where $\tau = L/c$ as before and

$$\gamma \equiv \frac{r - \frac{1}{2}}{r + \frac{1}{2}} = \frac{2r-1}{2r+1}.$$

It is shown below that a zero of transmission will occur in the LHP for all positive resistances $r$. A zero in the right half-plane will occur only when the second impulse is greater in area than the first impulse; this condition corresponds to a negative resistance $r$.

The system function for this network (i.e., the Laplace transform of (4) with $z=e^{-p\tau}$) is given by

$$H_2(z) = \frac{E_{\text{out}}}{E_{\text{in}}} = \frac{1}{2}z\{1 + \gamma z^2\}. \qquad (5)$$

The impulse reponse described by (4) is plotted in Fig. 2(b) for $r=\infty$, $r=\frac{1}{2}$, and $r=0$. Note that for $r<\frac{1}{2}$ ohm, the second impulse changes sign but its area can never exceed that of the first impulse. If $r$ is negative but greater than $-\frac{1}{2}$ ohm, then the area of the second impulse is given by

$$\frac{\gamma}{2} = \frac{1}{2}\frac{2r+1}{2r-1} \text{ volt-seconds} \qquad 0 > r > -\frac{1}{2} \qquad (6)$$

and, as shown in Fig. 2(c), is greater than the first impulse. $H_2(z)$ can be rewritten as

$$H_2(z) = \frac{1}{2}z\left(1 - \left[\frac{2r+1}{1-2r}\right]z^2\right) \qquad -\frac{1}{2} < r < 0. \qquad (7)$$

If the two networks shown in Figs. 1 and 2 are placed in tandem but separated by an ideal isolator as shown in Fig. 3(a), the overall system function $H_{OA}(z)$ can be written using (2) and (7) as

$$H_{OA}(z) = H_1(z)H_2(z) = \left(\frac{1-\Gamma^2}{2}\right)z^2 \cdot \frac{1 - \left(\frac{2r+1}{1-2r}\right)z^2}{1-\Gamma^2 z^2} \qquad (8)$$

where

$$0 < \Gamma^2 < 1, \qquad \frac{2r+1}{1-2r} > 1.$$

The term $[(1-\Gamma^2)/2]$ in (8) is a constant while the term $z^2$ represents a time delay. Letting $A=(1-\Gamma^2)/2$, one obtains from (8)

$$H_{OA}(z) = Az^2 \cdot \frac{1 - \exp\left\{\left[\ln\left(\frac{2r+1}{1-2r}\right) - 2p\tau\right]\right\}}{1 - \exp\left[\ln\Gamma^2 - 2p\tau\right]}. \qquad (9)$$

Note that a zero of transmission exists when

$$p_{\text{zero}} = \frac{1}{2\tau}\ln\left(\frac{2r+1}{1-2r}\right) \pm j\frac{n\pi}{\tau} \qquad (10)$$

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[1] W. Steenaart, "A contribution to the synthesis of distributed all-pass networks," *Generalized Network Symp. Rec.*, April 1966.

[2] H. Van DeVaart, "Pulse compression using X-band magnetoelastic waves in YIG rods," *Proc. IEEE (Letters)*, vol. 54, pp. 1007–1008, July 1966.

[3] W. Gongianni and J. Harrington, "Ultrawide bandwidth pulse compression in YIG," *Proc. IEEE (Letters)*, vol. 54, pp. 1074–1075, August 1966.

[4] J. G. Ferguson and C. W. Barret, "Stripline all-pass networks for pulse compression," *Symp. on Pulse Compression Techniques and Applications*, TRR-62-580, April 1963.

[5] G. F. Ross, "The transient behavior of certain TEM mode 4-port networks," *IEEE Trans. Microwave Theory and Techniques*, vol. MTT-14, pp. 528–542, November 1966.

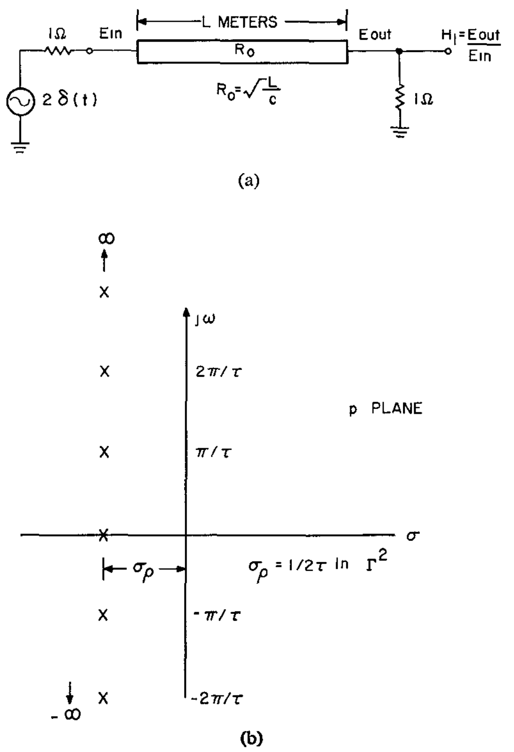



Fig. 1. (a) Network #1. (b) The pole-zero plot of the network.

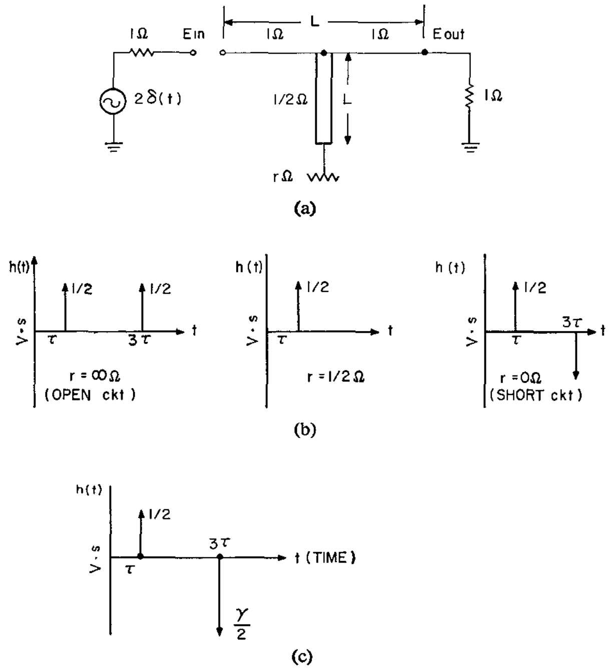



Fig. 2. (a) Network #2. (b) Impulse response as a function of $r$. (c) Impulse response for negative $r$ where $0 > r > -\frac{1}{2}$ ohm.

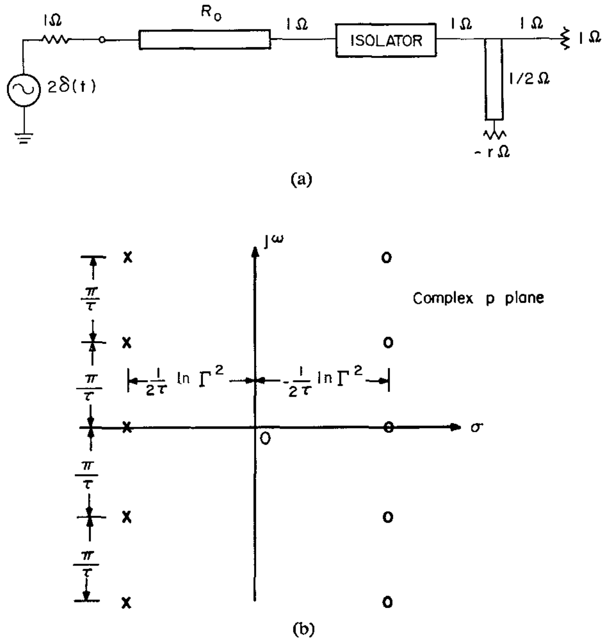



Fig. 3. (a) Overall network. *Note:* driving point impedance of an isolator is 1 ohm over the band, and the resistance $r$ is maintained over the band. (b) Pole-zero pattern of the network.

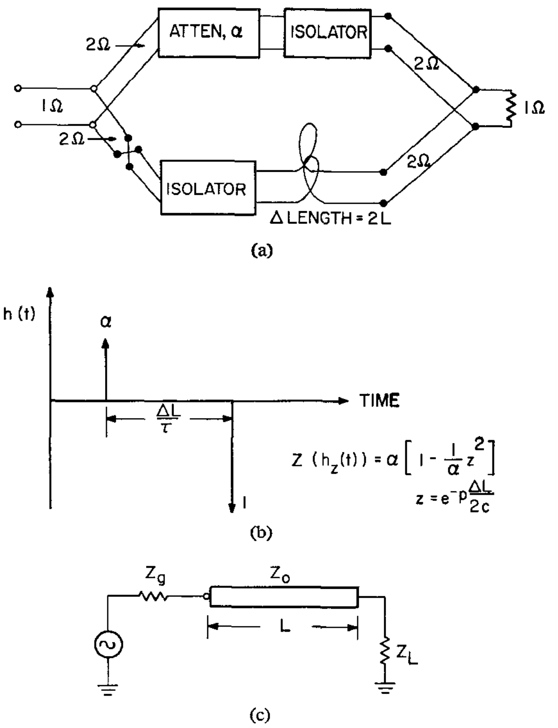



Fig. 4. Alternate passive solution using isolators. (a) Network #2. (b) Impulse response. (c) Modified network #1.

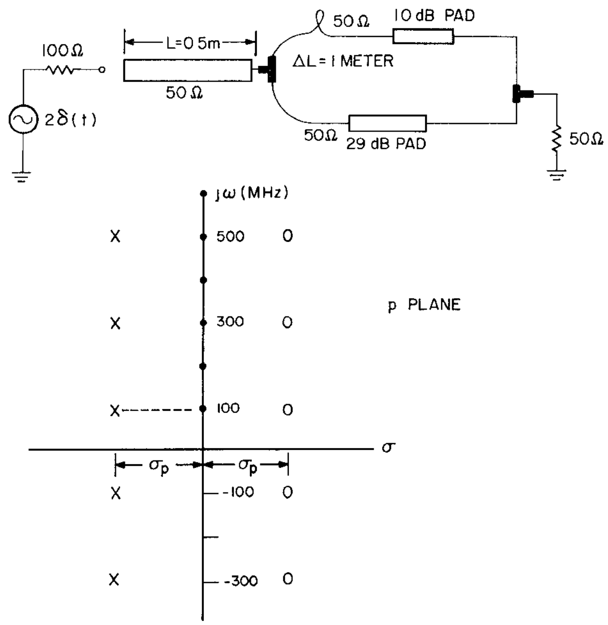



Fig. 5. Alternate passive solution using attenuators. *Note:*

$$|\sigma_p| = \frac{1}{2\tau_1} \ln \Gamma^2 = \frac{0.954}{2\tau_1} = \frac{0.954}{\tau_2} = 1.91 \times 10^8$$

where

$$\tau_1 = \sqrt{\epsilon}\,\frac{L}{\tau} = \sqrt{\epsilon}\,\frac{0.5}{3} \times 10^{-8} = \sqrt{2\ 26} \times 1.67 \times 10^{-3}|_{\text{polyethylene}} = 2.5 \times 10^{-9}\ \text{second}$$

$$2\tau_1 = 5\ \text{n} \qquad \tau_2 = 5\ \text{ns}$$

$$\text{Complex pole-zero pairs occur at } f_0 = \frac{1}{4\tau_1}, \frac{3}{4\tau_1}, \cdots, \left(\frac{2k-1}{4\tau_1}\right) = 100,\ 300,\ 500\ \text{MHz, etc.}$$

while a pole exists when

$$p_{\text{pole}} = \frac{1}{2\tau} \ln \Gamma^2 \pm j\frac{n\pi}{\tau}$$
$$n = 0, 1, 2, \cdots. \qquad (11)$$

If $\Gamma^2$ is selected to satisfy the relationship

$$\Gamma^2 = \frac{1-2r}{1+2r} \qquad (12)$$

then a zero of transmission exists when

$$\text{Re}\,[p_{\text{zero}}] = \frac{1}{2\tau} \ln \frac{1}{\Gamma^2} = -\frac{1}{2\tau} \ln \Gamma^2 \qquad (13)$$

while a pole exists at

$$\text{Re}\,[p_{\text{pole}}] = \frac{1}{2\tau} \ln \Gamma^2. \qquad (14)$$

This pole-zero pattern is shown in Fig. 3(b). It is easy to show geometrically, that like the constant resistance lattice this network is an allpass phase equalizer whose amplitude spectrum or CW response is flat for *all* frequencies. In practice one would use this filter to equalize broadband signals having their principle spectral components in the vicinity of the pole-zero pair

$$Pz_1 = -\frac{1}{2\tau} \ln \Gamma^2 \pm j\frac{\pi}{\tau},$$
$$Pp_1 = +\frac{1}{2\tau} \ln \Gamma^2 \pm j\frac{\pi}{\tau}. \qquad (15)$$

This network is thus very broad band, providing the negative resistance $r$ can be maintained over the required band of interest.

A more desirable solution to this problem and one that does not involve the use of negative resistance devices can be accomplished by replacing network #2 with the one shown in Fig. 4(a) and (b) where two additional isolators and crossed wires are added. The system function for this configuration is proportional to

$$\hat{H}_2(z) \sim \left(1 - \frac{1}{\alpha} z^2\right) \qquad (16)$$

and is of the same form as (7). If the wires are uncrossed to reduce construction complexity. the system function is proportional

$$\hat{H}_2(z) \sim \left(1 + \frac{1}{\alpha} z^2\right). \qquad (17)$$

The plus sign in (17) necessitates that network #1 also be modified. This modification is required to assure that a positive sign also exists between terms in the denominator of $H_1(z)$. The modified circuit for network #1 is shown in Fig. 4(c). The system function for this network is

$$\hat{H}_1(z) = \frac{(1+\Gamma_1)(1+\Gamma_2)z}{1+\Gamma_1\Gamma_2 z^2} \qquad (18)$$

where

$$\Gamma_1 \equiv \frac{Z_0 - Z_g}{Z_0 + Z_g},$$

$$\Gamma_2 \equiv \frac{Z_L - Z_0}{Z_L + Z_0}.$$

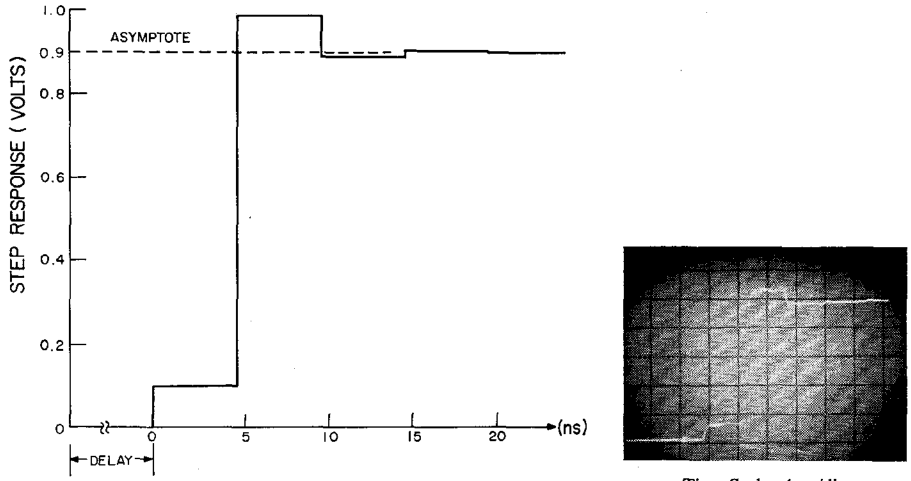
To obtain a positive sign in the denominator of (18), $\Gamma_1 \Gamma_2$ must be positive; thus, either

$$Z_g < Z_0 < Z_{\text{load}} \qquad (19a)$$

or

$$Z_g > Z_0 > Z_{\text{load}}. \qquad (19b)$$

It is also possible to eliminate the need for isolators by the proper interconnection of the two networks and the introduction of sufficient loss. For example, consider the overall network shown in Fig. 5 where the network #1 satisfies the condition imposed by (19b). The isolator between networks #1 and #2 can be removed because network #1 is terminated in its required mismatch (e.g., 2–50 ohm lines in parallel or 25 ohms): the isolators in each parallel path of the network shown in Fig. 5(a) can be replaced by 10 dB attenuators which provide sufficient loss to absorb most of the unwanted reflections between the tees. The overall system function of the network shown in Fig. 5 is given approximately by




(a)

Time Scale: 4 ns/div
(b)

Fig. 6. The step response of the equalizer. (a) Theoretical step response. (b) Measured response.

$$\hat{H}_{0A}(z) \doteq \hat{H}_1(z)\hat{H}_2(z),$$

$$= \left[\frac{(1+\Gamma_1)(1+\Gamma_2)z}{1+\Gamma_1\Gamma_2 z^2}\right] \cdot \left[\frac{2}{3}\cdot\frac{\alpha}{(3.15)}\left(1+\frac{1}{\alpha}z^2\right)z^l\right],$$

$$= Bz^{l+1}\left[\frac{1+\dfrac{1}{\alpha}z^2}{1+\alpha z^2}\right],$$

$$B = \frac{2(1+\Gamma_1)(1+\Gamma_2)\Gamma_1\Gamma_2}{(3)(3.15)},$$

$$\alpha = \Gamma_1\Gamma_2. \qquad (20)$$

The *differential* attenuation required between lines in network #2 is equal to the ratio of $1/\Gamma^2 = 1/9$ [see (10) and (13)]: a voltage reduction factor of about 1/9 is achieved by the use of a 19 dB pad. The first pole-zero pair occurs when the magnitude of the real part of $p$ equals

$$|\operatorname{Re} p| = \left|\frac{1}{2\tau}\ln\frac{1}{\Gamma^2}\right| = \frac{0.952}{2\tau_1} = \frac{0.952}{\tau_2} \qquad (21)$$

and

$$|\operatorname{Im} p| = \frac{\pi}{2\tau_1} = \frac{\pi}{\tau_2} = 2\pi f_0$$

$$f_0 = \frac{1}{4\tau_1} = \frac{1}{2\tau_2}.$$

If

$$\tau_1 = 2.5 \times 10^{-9} \text{ seconds}$$

then

$$f_0 = 100 \text{ MHz}$$

$$\sigma = \frac{0.952}{5 \times 10^{-9}} = 0.191 \times 10^9. \qquad (22)$$

$\tau_1 = 2.5\times10^{-2}$ seconds corresponds to a line length of

$$L_1 = \frac{c\tau_1}{\sqrt{\epsilon}} = \frac{3\times10^8\times2.5\times10^{-9}}{2.26} = 0.5 \text{ meter}$$

$$\Delta L = 2L_1 = 1 \text{ meter} \qquad (23)$$

where the dielectric constant $\epsilon$ equals 2.26 for polystyrene.

The overall loss coefficient of the network is $B$. From (20)

$$B = \frac{2(1+\Gamma_1)(1+\Gamma_2)\Gamma_1\Gamma_2}{(3)(3.15)} = \frac{2\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{1}{9}\right)}{(3)(3.15)} = \frac{1}{94}$$

$$= 39.5 \text{ dB loss}$$

$$\text{(independent of frequency)} \qquad (24)$$

The network shown in Fig. 5 was fabricated in the laboratory. It was excited by a unit step generator and the output displayed on a sampling oscilloscope. The theoretical step response was obtained by multiplying (20) by $1/p$, dividing numerator by denominator and inverting the resulting expression term by term and is plotted in Fig. 6(a); a measurement of the actual step response is shown in Fig. 6(b). It can be seen that the results are in close agreement with the theory.

In conclusion, it should be noted that the network can readily be scaled to make, for example, the first pole-zero cluster occur at 1 GHz. Here the line length $L$ must be reduced by a factor of ten i.e., $L = 0.05$ meter. The network loss can be reduced by using wide band isolators and/or a TWT amplifier centered appropriately in the band of interest.

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## Mounted Diode Equivalent Circuits

The use of semiconductor diodes at microwave frequencies continues to arouse interest in practical circuit characterization for mounted diodes.

Accurate wideband circuit description and evaluation techniques are needed to design components for predictable performance or optimize component response, and are essential if the design work is to be done by a computer on components employing widely spaced frequencies and wide bandwidths such as occur with parametric amplifiers and multipliers.

Previously Getsinger[1] gave theoretical background on a conceptual scheme for describing microwave frequency behavior of packaged diodes and mounts by the use of lumped-element equivalent circuits. Separation of the circuit of the packaged diode from the circuit of the mount was shown to be possible if the equivalent circuit of the packaged diode described the radial-line TEM-mode impedance found at a terminal surface defined to be at the outer diameter of the cylindrical diode package.

The purpose of this correspondence is to extend the work mentioned above by pointing out that the same point of view can be used even when the diode is unpackaged, that eliminating the diode package does not eliminate the coupling-network problem, that a more rigorous consideration of the diode mounting situation indicates that the diode equivalent circuit and mount equivalent circuit are not completely separable although the common elements are usually very small and can be measured, and finally that the approach of the referenced paper[1] holds even when the diode is not mounted between parallel metal planes, as was specified therein, provided mount parameters can be measured with the diode in place.

*The basic diode coupling problem is to relate the electromagnetic fields across macroscopic terminal surfaces of a uniform transmission line to the fields within a microscopic region (the diode junction) of space.*

In the preceding work[1] it was pointed out that the solution to this problem could usually be stated in terms of lumped-element circuit mathematics, and thus, in terms of an equivlent circuit or coupling network between the uniform-line terminal surface and the diode junction.

If one considers the total equivalent circuit of a packaged diode mounted in a useful way, and then mentally removes the packaging, it can be seen that the circuit configuration does not change; only the values of some circuit elements are changed. Thus, the circuit problem may not be perceptibly simpler without a package than with one. In fact, it is often helpful to define a cylindrical enclosure for the unpackaged mounted diode and treat it as though it were packaged.

For example, the circuit of Fig. 1 holds for a diode mounted in a waveguide, whether or not the diode is packaged. If packaged, the

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[1] W. J. Getsinger, "The packaged and mounted diode as a microwave circuit," *IEEE Trans. Microwave Theory and Techniques*, MTT-14, pp. 58–69, February 1966.